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# ON THE CONVERGENCE OF THREE ITERATIVE FFT-BASED METHODS FOR COMPUTING THE MECHANICAL RESPONSE OF COMPOSITE MATERIALS

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The last decade has witnessed a growing interest for the so-called "FFT-based methods" for computing the overall and local properties of heterogeneous materials submitted to mechanical solicitations. Since the original method was introduced by Moulinec and Suquet [1], several authors have proposed different algorithms to better deal with non-linear materials or with materials whith highly contrasted mechanical properties between their constituents. The present paper aims to compare three methods of this family of algorithms which were designed to accelerate the convergence of the scheme.

The study concerns a linear elastic material - although the methods involved can be extended into the case of non-linear behavior - submitted to a prescribed overall strain $\boldsymbol{E}$. The stiffness tensor $\boldsymbol{c}(\boldsymbol{x})$ of the material varies with the position $\boldsymbol{x}$. The numerical method proposed by Moulinec & Suquet lies on the iterative resolution of the Lippmann-Schwinger equation and can be summarized by the following relation between two successive iterates $\boldsymbol{\varepsilon}^{\mathrm{i}}$ and $\boldsymbol{\varepsilon}^{\mathrm{i+1}}$ of the strain field:

$$\boldsymbol{\varepsilon}^{\mathrm{i+1}}(\boldsymbol{x}) = -\boldsymbol{\Gamma}^0 * \big((\boldsymbol{c}(\boldsymbol{x}) - \boldsymbol{c}^0) : \boldsymbol{\varepsilon}^{\mathrm{i}}(\boldsymbol{x})\big) \quad + \boldsymbol{E} \tag{1}$$

where $\boldsymbol{c}^0$ is the stiffness tensor of a reference medium supposed to be linear elastic, where $\boldsymbol{\Gamma}^0$ is a Green operator associated to $\boldsymbol{c}^0$ and where $*$ denotes the convolution operator. The number of iterations necessary to reach a given tolerance is in the order of magnitude of the contrast between the mechanical properties of the material phases.

Eyre & Milton [2], Michel et al. [3] and Monchiet & Bonnet [4] proposed different schemes in order to accelerate the convergence of the initial scheme. The scheme of Monchiet & Bonnet can be suitably rewritten as:

$$(\boldsymbol{c} + \boldsymbol{c}^0) : \boldsymbol{\varepsilon}^{\mathrm{i+1}} = (\boldsymbol{c} + \boldsymbol{c}^0) : \boldsymbol{\varepsilon}^{\mathrm{i}} - \alpha \boldsymbol{c}^0 : \boldsymbol{\Gamma}^0 * \boldsymbol{c} : \boldsymbol{\varepsilon}^i - \beta \boldsymbol{\Delta}^0 * \boldsymbol{\varepsilon}^i \; - \; \beta \boldsymbol{c}^0 : (< \boldsymbol{\varepsilon}^{\mathrm{i}} > - \boldsymbol{E}) \quad . \tag{2}$$

It has been recently demonstrated [5] that the scheme of Eyre & Milton and the augmented Lagrangian scheme of Michel et al. are particular cases of the scheme of Monchiet & Bonnet with $\alpha = \beta = 2$ and $\alpha = \beta = 1$ respectively. A convenient upper bound $\mathcal{R}_u$ of the spectral radius of the iterative operator can be exhibited, from which sufficient conditions of convergence of the scheme (2) are derived. The value of the reference medium $\boldsymbol{c}^0$ that minimizes $\mathcal{R}_u$ can be determined. Moreover, it can be demonstrated that the scheme of Eyre & Milton is the one that minimizes $\mathcal{R}_u$. These results are illustrated in figure (1).

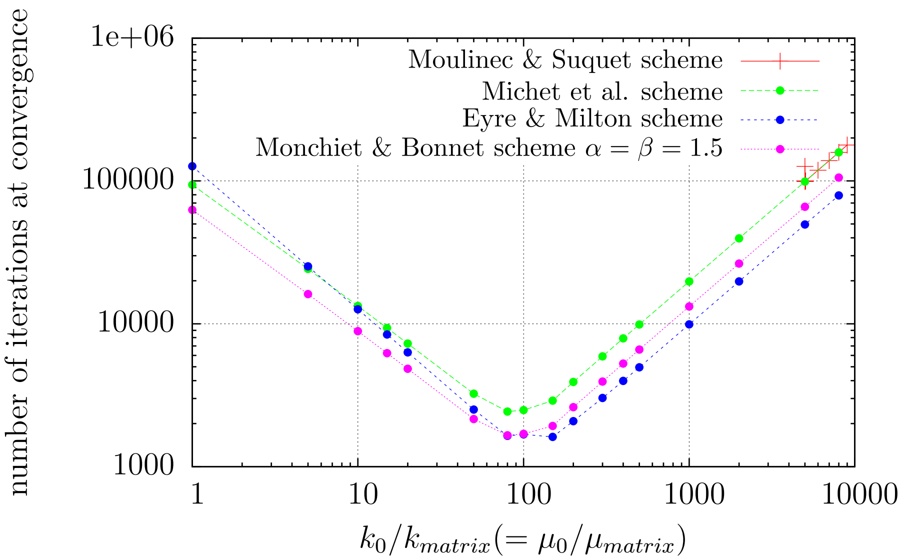


Figure 1: Number of iterations at convergence (tolerance $= 10^{-10}$) for different choices of reference material $\boldsymbol{c}_0$, for a contrast of 10000 between the mechanical properties of inclusion and matrix of a given microstructure.

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